



Hydraulic structures. Dams and reservoirs

Concrete dam engineering -2

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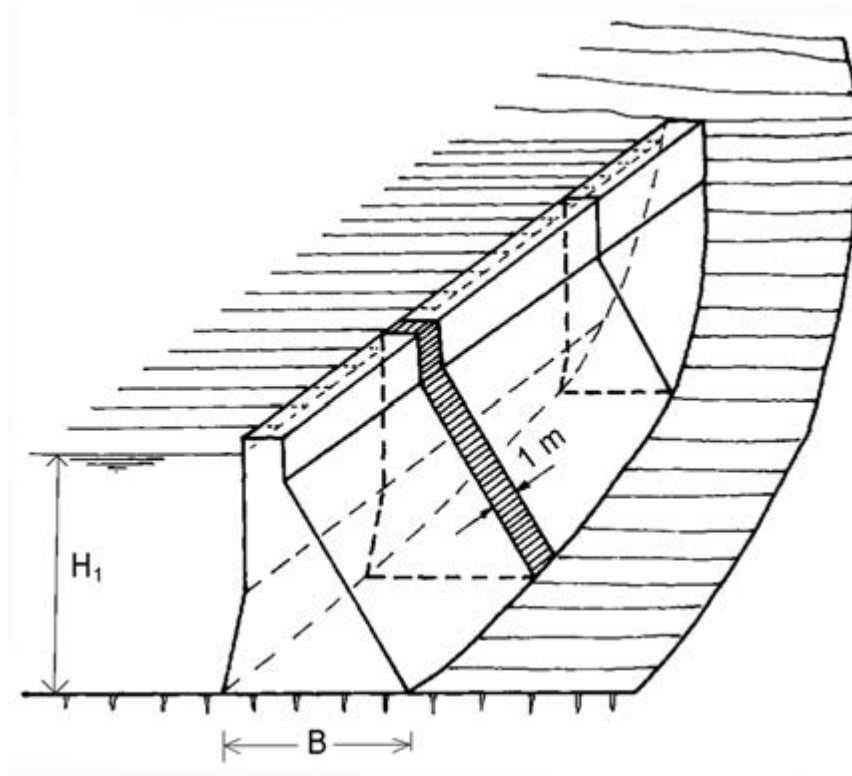
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**Strengthening of master curricula in water resources
management for the Western Balkans HEIs and stakeholders**

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Stress and Stability Analysis

Limiting state method



Modes of Failure and Criteria for Structural Stability of Gravity Dam

A gravity dam may fail in the following ways :

- (1) By crushing.
- (2) By development of tension, causing ultimate failure by crushing.
- (3) By shear failure called sliding.
- (4) By overturning (or rotation) about the toe.

The failure may occur at the foundation plane (*i.e.* at the base of the dam) or at any other plane at higher level.

COMPRESSION OR CRUSHING

A dam may fail by the failure of its materials.

The compressive stress may exceed the allowable stress and the dam material may get crushed.

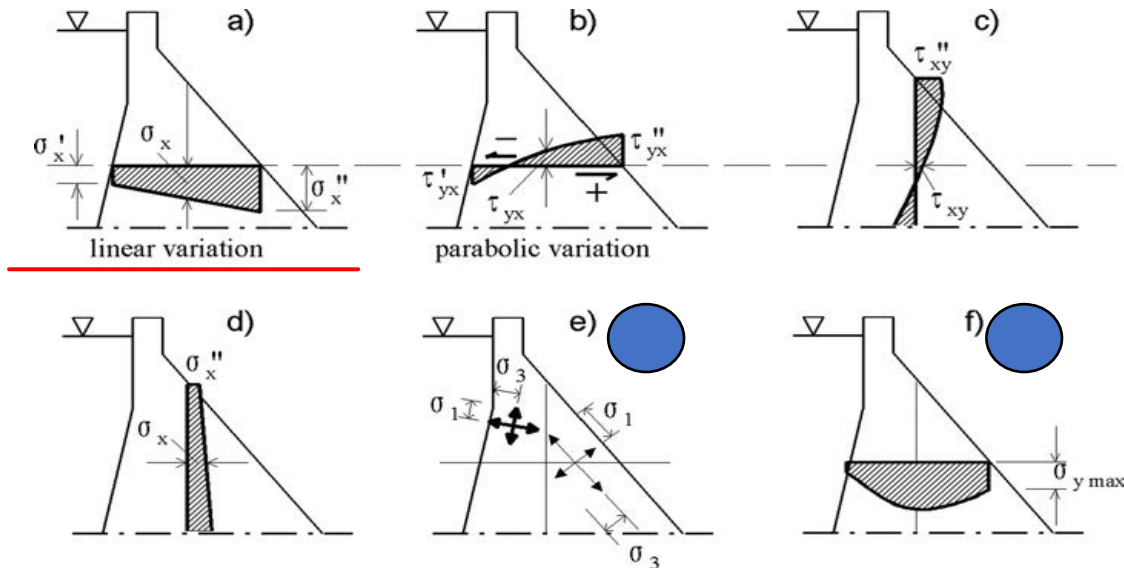
TENSION

Masonry and concrete gravity dam are usually designed in such a way that no tension is developed anywhere, because the materials can not withstand sustained tensile stresses.

If it subjected to such stresses, these materials may crack.

analysis of stresses

The **gravitational method** for the analysis of stresses is simple and is based on assumptions from the theory of elasticity. This method assumes that the 'trapezium law' is valid; that is to say, it assumes that **normal stresses vary linearly between the upstream and the downstream face in all horizontal planes**.



A dam may fail by the failure of its materials, *i.e.* the compressive stresses produced may exceed the allowable stresses, and the dam material may get crushed. **The vertical direct stress distribution at the base is given by the equation**


$$\sigma_y = \text{Direct stress} + \text{Bending stress}$$

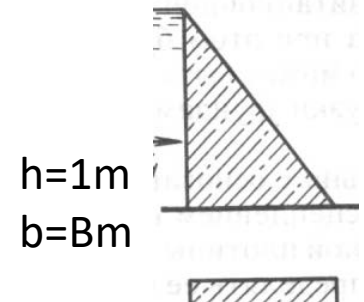
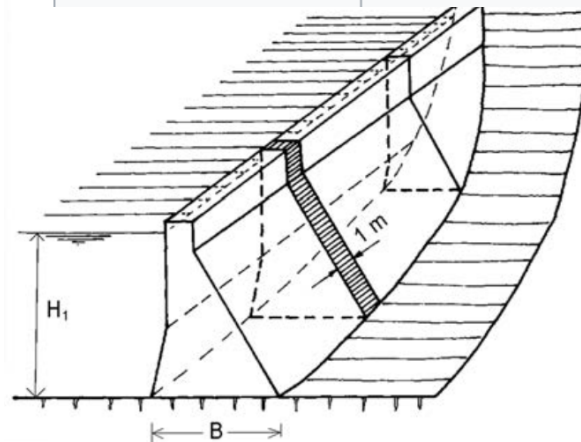
Vertical normal stress σ_y in an arbitrary plane, according to this method, is determined by the expression for eccentric stresses for beams, along with certain modifications:

Where W is Section modulus

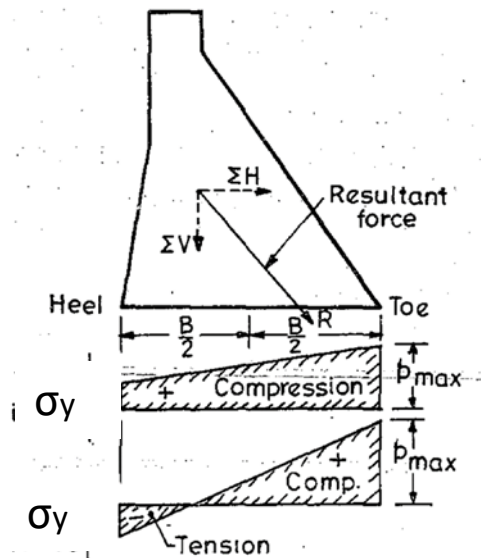
$$\sigma_{y_{1,2}} = \frac{\sum V}{F} \pm \frac{\sum M}{W} \rightarrow$$

$$\sigma_{y_{1,2}} = \frac{\sum V}{B} \pm \frac{6 \sum M}{B^2}$$

Cross-sectional shape	Figure	Equation
Rectangle		$S = \frac{bh^2}{6}$



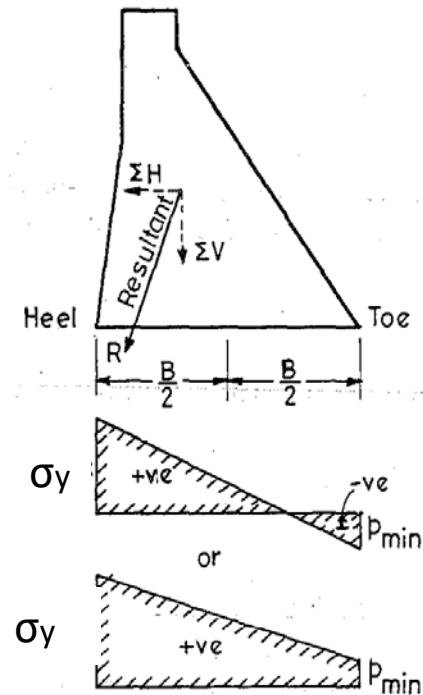
Vertical Stress Distribution for Reservoir Full case



$$\sigma_{y1,2} = \frac{\Sigma V}{B} \mp \frac{6 \Sigma M}{B^2}$$

1-Heel
2-Toe

Vertical Stress Distribution for Empty Reservoir case

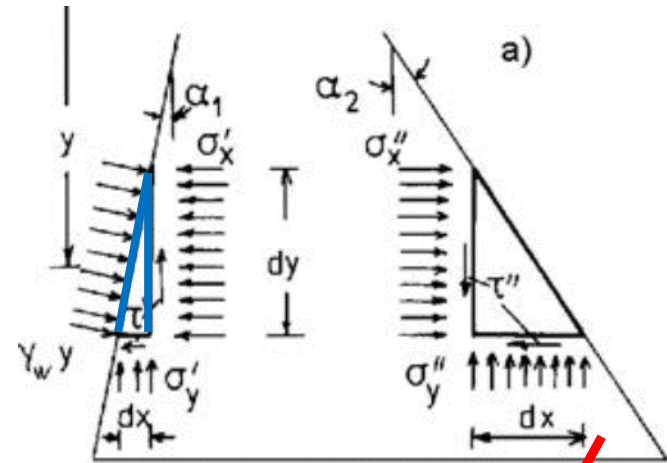


The normal horizontal stresses σ_x and the shear stresses $\tau_{xy} = \tau_{yx} = \tau$, can be determined

from the conditions for equilibrium of the elementary triangles with cathetus dx and dy , extracted from the slopes in the cross-section of the dam:

the condition for equilibrium:
the projection of all forces acting on the triangle over y -axis equal to zero, we obtain:

$$\gamma_w y \frac{dx}{\sin \alpha_1} \sin \alpha_1 - \tau' dy - \sigma'_y dx = 0$$



$$\tau = (\gamma_w y - \sigma'_y) \tan \alpha_1$$

$$\sigma'_x = \gamma_w y - (\gamma_w y - \sigma'_y) \tan^2 \alpha_1$$

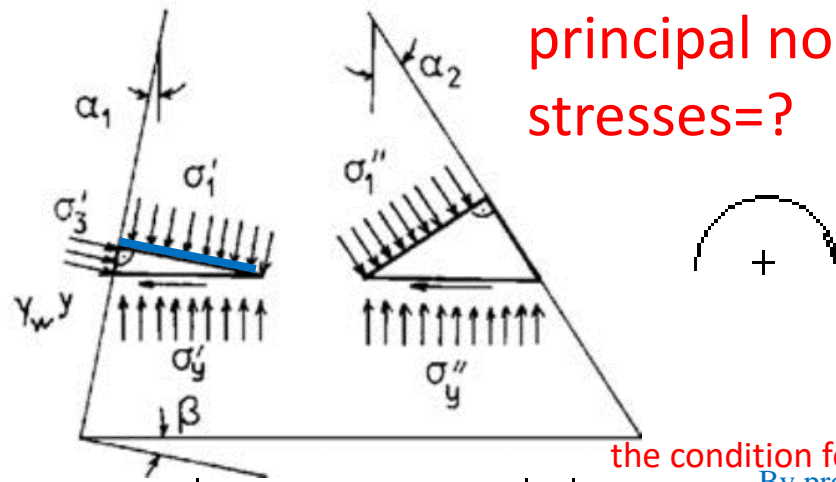
In the same way, the downstream slope is obtained:

$$\tau'' = \sigma''_y \tan \alpha_2$$

$$\sigma''_x = \sigma''_y \tan^2 \alpha_2$$



principal normal stresses=?



the condition for equilibrium

By projecting along the y-axis of all forces acting on the triangle in the upstream face, one obtains:

Principal coordinate system is rotated in relation to the x-y coordinate system for an angle β ,

$$2\beta = -\frac{2\tau}{\sigma_x - \sigma_y}$$

$$\sigma_{1,3} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau}$$

$$\tau_{\max} = \pm \frac{1}{2}(\sigma_1 - \sigma_3)$$

$$\gamma_w y dx \sin \alpha_1 \sin \alpha_1 + \sigma'_3 dx \cos \alpha_1 \cos \alpha_1 - \sigma'_y dx = 0$$

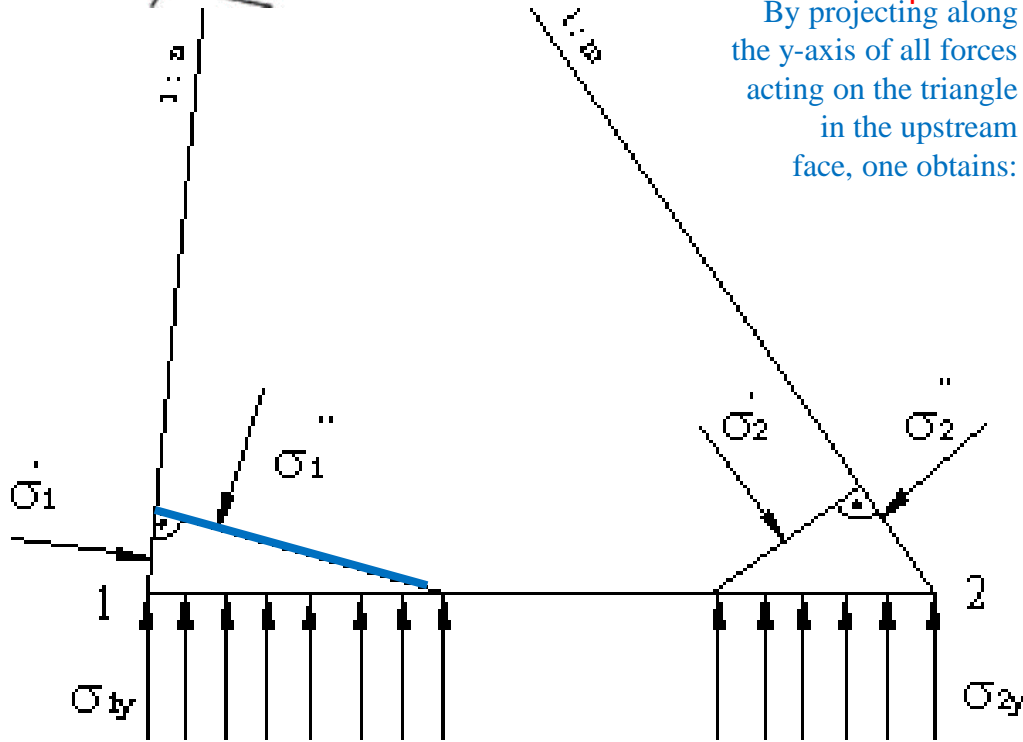
principal normal stresses

$$\sigma_1^I = \gamma_E \cdot h$$

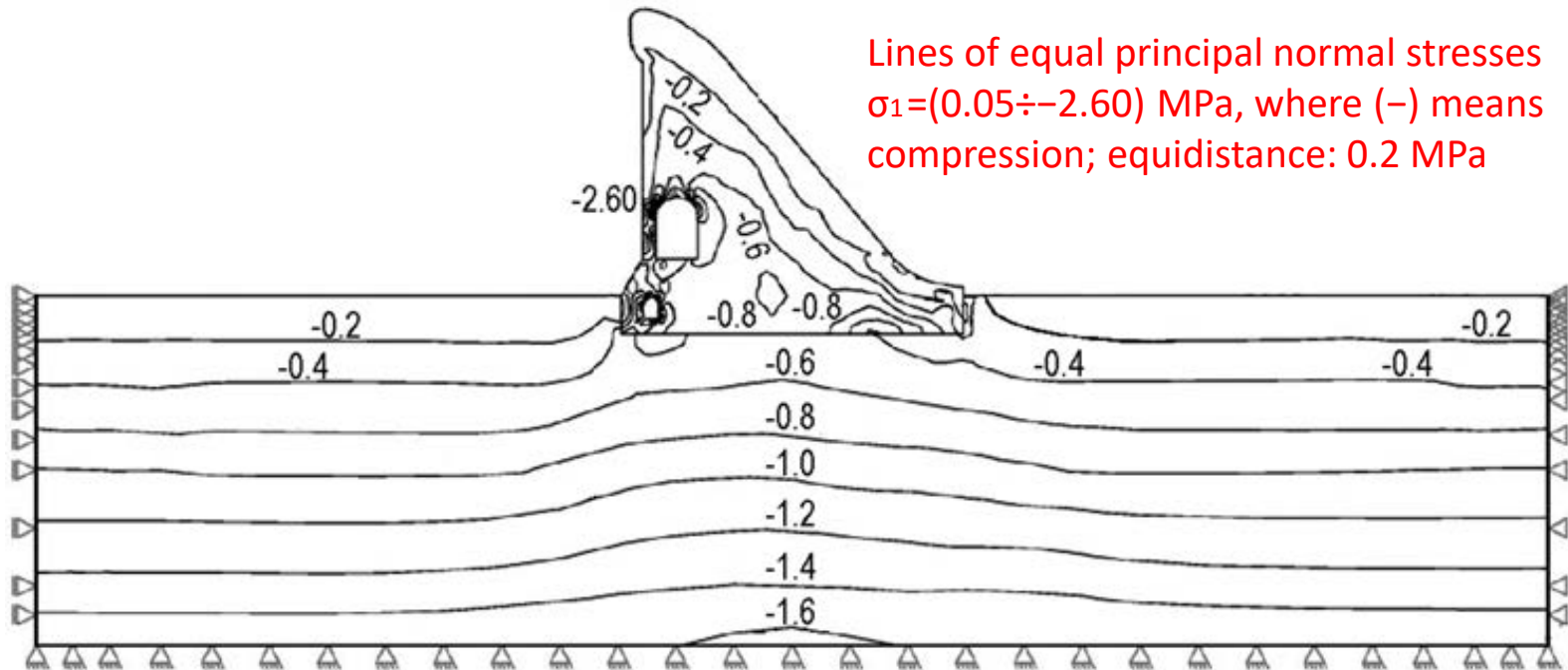
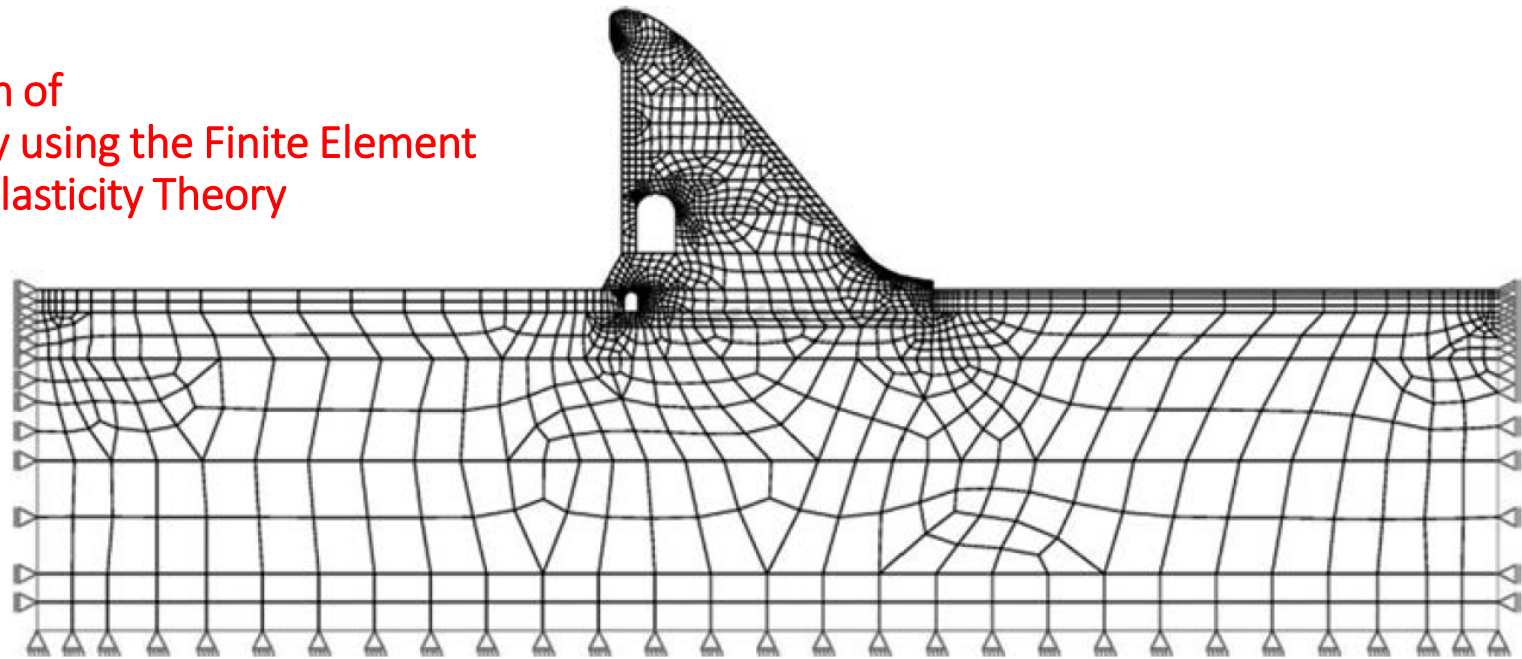
$$\sigma_1^{II} = \sigma_{y1} \cdot (1+n^2) - \gamma_E \cdot h \cdot n^2$$

$$\sigma_2^I = \sigma_{y2} \cdot (1+m^2)$$

$$\sigma_2^{II} = 0$$



Calculation of stresses by using the Finite Element Method, Elasticity Theory



Lines of equal principal normal stresses $\sigma_1=(0.05\div-2.60)$ MPa, where (-) means compression; equidistance: 0.2 MPa

Loading capacity

$$n_c \sigma \leq \frac{m}{k} R$$

σ -principal stress

R- prismatic strength

$$n_c N \leq \frac{m}{k} R$$

1st limiting state :

The values of loads and stresses which develop in the structure or its foundation *must not exceed* the load-resisting capacity of the structure or foundation!

N - design generalized load

/it means forces or moments or stresses/

R - design generalized load-resisting capacity

nc - loading combination coefficient

/ for usual combo:1.0 for unusual combo:0.9

for End of Construction:0.95/

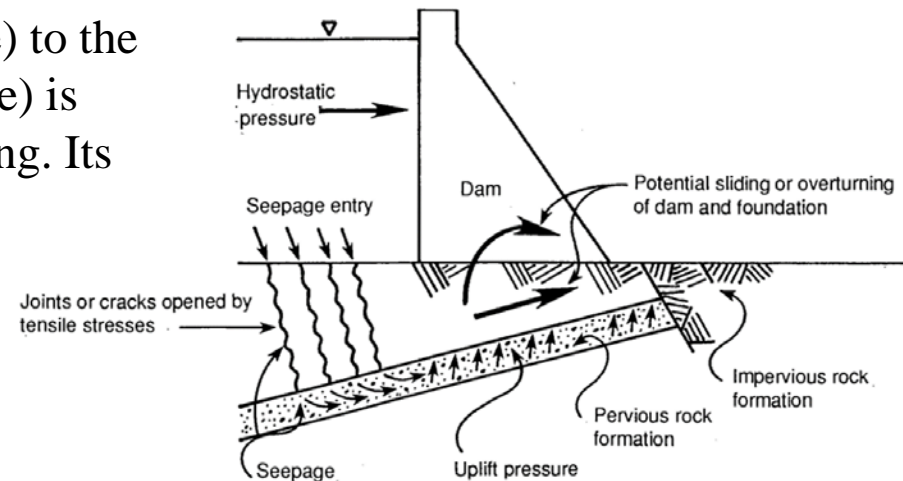
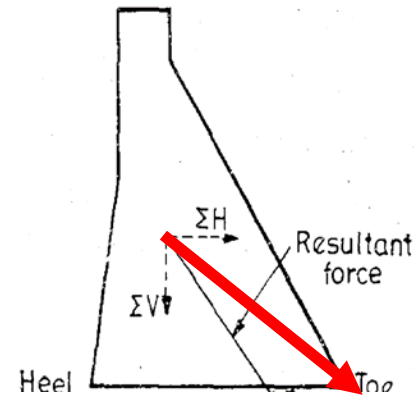
m - working condition coefficient – vary in wide limits

k - safety factor, it takes into account degree of responsibility,

/ for the First Class or category of the structure is 1.25/

Overturning stability

If the resultant of all forces acting on a dam at any of its sections, passes outside the toe, the dam shall rotate and overturn about the toe. Practically, such a condition shall not arise, as the dam will fail much earlier by compression. The ratio of the righting moments about toe (anti clockwise) to the over turning moments about toe (clock-wise) is called the factor of safety against overturning. Its value, generally varies between 2 to 3.



Sliding stability

$$\text{F.S.S. (Factor of safety against sliding)} = \frac{\mu \cdot \Sigma V}{\Sigma H}$$

The friction developed between two surfaces is equal to $\mu \Sigma V$, where ΣV is the algebraic sum of all vertical forces, and μ is the coefficient of friction between the two surfaces. The value of μ generally varies from 0.65 to 0.85.

